Insights into practical reasoning from financial crises.

January, 2018
A contribution to the CRUISSE Network Conference,
21-22 September 2017.

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Abstract

Financial markets are dominated by radical uncertainty. In such circumstances, decision-makers cannot rely on a mathematical/statistical models being representative of the future economy. These observations do not preclude the use of mathematical models in finance. Rather, they highlight Locke's distinction of mathematics employed in physica to that employed in practica, which existed right through to the mid-nineteenth century. This talk will explain how the distinction has been manifest in the history of mathematics, is present in contemporary financial mathematics and, with reference to actual events, how the distinction, when employed, has contributed to profitability during financial crises.

The commercial origins of mathematical probability

The origin mathematical probability is in a problem presented by Thomas Aquinas (1225-1274) in his *Summa Theologica*. In the 'Second part of the Second part', the *Summa* addresses sins committed in buying and selling. The first article asks the question 'Whether it is lawful to sell something for more than it is worth?' and Aquinas's answer is that, as long as there is no fraud involved and an equality between what is being exchanged is established, then there is no ethical problem (Aquinas, 1947, pp. Q77,1). Within this context, Aquinas discussed the specific question of 'Whether the seller is bound to state the defects of the thing sold?' and discusses a well-known problem from stoic philosophy.

> A grain merchant from Alexandria arrives at Rhodes, which is gripped by famine. The merchant knows that other merchants are following him with plentiful supplies of grain, though the town's inhabitants do not know this. How should the merchant price the grain he has?

The classical argument was that the merchant should not charge the 'market price', given the knowledge of more supplies coming. Aquinas disagreed.

> in the case cited, the goods are expected to be of less value at a future time, on account of the arrival of other merchants, which was not foreseen by the buyers.

> Wherefore the seller, since he sells his goods at the price actually offered him, does not seem to act contrary to justice through not stating what is going to happen. If, however, he were to do so, or if he lowered his price, it would be exceedingly virtuous on his part: although he does not seem to be bound to do this as a debt of justice. (Aquinas, 1947, pp. Q77,3)

Aquinas' justification is based on the observation that while the merchant may *believe* there are more grain shipments on the way, they do not *know*; the future is uncertain and this uncertainty creates the opportunity for profit (Rothbard, 1996, p. 53).

Aquinas expanded on his argument

> because the just price of things is…not fixed with mathematical precision, but depends on a kind of estimate, … so that a slight addition or subtraction would not seem to destroy the equality of justice (Aquinas, 1947, pp. Q77,1)

Kaye (Kaye, 1998, pp. 98-99) makes the point that Aquinas had separated the 'just price' − determined by divine law − from the 'market price' − established by human judgement − emphasising that the just price is not a single, fixed point, but rather it defines a fluctuating range of possible prices. So long as the 'market price', agreed by merchants, is within the bounds of what is 'just' the merchants are not contravening natural law. This was theologically important because if there was no room for judgement in pricing then the individual's moral responsibility in commerce was removed suggesting merchant's need not exercise moral choice.

Another Scholastic, the 'Spiritual Franciscan' Pierre-Jean Olivi (c.1248-1298), disagreed with Aquinas' position. Olivi argued that the metaphysical probability of more grain arriving in Rhodes had a certain reality, which Aquinas was ignoring by focusing on the 'physical' reality of the market prices (Kaye, 1998, p. 119), (Franklin, 2001, pp. 265-267). Olivi realised that, fundamentally, market exchange was about equating expectations. At this point he makes an important conceptual leap: since these expectations where expressed as quantified prices the implication was that probability, itself, was quantifiable.

Both Aquinas and Olivi worked in a Scholastic framework founded on Aristotle and this context highlights the significance of Olivi's observation. Their economic analysis was based on interpretations of *Nicomachean Ethics*, where Aristotle discussed under what conditions economic exchange was just. Aristotle was working within a framework constructed by Plato wherein 'justice' is defined as the virtue that ensures a functionally differentiated system, such as a society, works well (Plato, 1969, pp. 4.434a-c). Aristotle argued that because society was complex, different people had different capabilities and needs, and so exchange was an essential component of 'justice' and bound society together. Central in this conception of exchange, in ensuring social cohesion, was that there needed to be equality in what was exchanged. Aristotle observed, “there is no giving in exchange” (Aristotle, 2011, pp. 1133a1-1133a5), it had to be a clearly reciprocal arrangement. The guiding principle of exchange could not be to generate a profit – because that would suggest an inequality – but ‘fairness’ (Aristotle, 2011, pp. 1133a15-1133a30), (Kaye, 1998, p. 51), (Judson, 1997). Aristotle summarised his position with the statement that “there would be no association without exchange, no exchange without equality and no equality without commensurability” (Aristotle, 2011, pp. 1133b15-1133b20). Because everything that was to be fairly exchanged needed to be equated it needed to be measured and money gave the ‘yard-stick’ of value (Aristotle, 2011, pp. 1133a19-20).

The observation that money offered a universal yard-stick caught the attention of Albert the Great (c. 1200-1280). In studying *Nicomachean Ethics*, Albert realised that when Aristotle used money to value he was creating a link between the object and the number. Aristotle had discussed measurement in *Physics* and *Metaphysics*, parts of the *Organon* written after *Nicomachean Ethics*, where he claimed that a measure shared the same Substance as the subject of measurement. For example, numbers are measured by the smallest number, ‘1’, distances are measured by the smallest length of distance, say an inch, while wine is measured by a unit of volume. Albert noticed that money was a very special measure in that it applied to all goods in a market, but did not (or only occasionally) share the Substance of the goods. If Aristotle was right about money being a measure he could not be right about a measure sharing the Substance of the measured. This realisation enabled Albert’s successors, starting with Olivi and then the ‘Merton Calculators’, to revolutionise European science by mathematicising it – chance was quantifiable and time and space were commensurable – in a way that other contemporary societies could not. (Crosby, 1997), (Kaye, 1998), (Henry, 2008, pp. 18-55), (Dear, 2009, pp. 64-78).

This Aristotelian, approach to probability as a moral science concerned with establishing equality in exchange, provided the basis of both Huygens’ *Van Rekeningh in Spelen van Geluk* (1657) and Bernoulli’s *Ars Conjectandi* (1713) (Sylla, 2003), (Sylla, 2006) and was evident in Laplace’s *Essai philosophique sur les probabilités* (1812) and Poisson’s *Recherches sur la probabilité des jugements en matiére criminelle et en matiére civile* (1837). The ‘frequentist’ approach to probability, rooted in gaming rather than of commercial exchange, began to eclipse the ‘moral’ approach after it was introduced by de Moivre and Montmort in the second decade of the eighteenth century. The disappearance of the moral approach to probability is evident in Todhunter’s seminal *A History of the Mathematical Theory of Probability from the time of Pascal to Laplace* of 1865. Todhunter effectively ignores Cardano’s 1564 discussion of probability, which was rooted in Aristotelian ethics (Bellhouse, 2005) and raised a problem with *Ars Conjectandi*, where Bernoulli happily discussed probabilities that did not sum to one; as illogical in the frequentist conception of probability.

Bernoulli began looking at probability around 1684 and between 1700 and his death in 1705, he worked on *Ars Conjectandi*, the first comprehensive textbook on probability. The *Ars*, written to explain how to 'measure probability', is made up of four: parts, a commentary on Huygens' *Van Rekeningh*, original work on calculating permutations and combinations; applications of these ideas to games of chance and finally the application of the ideas to "civil, moral and economic affairs" (Hald, 1990, pp. 220-224). In the first three parts, the *Ars* argues that 'chance', objective or aleatory probability, was determined by "equal conditions, honest dice, pure chance and symmetry" (Hald, 1990, p. 246). In the final part, Bernoulli introduced the concept of measuring subjective or epistemic probability. Believing in an omniscient God, with the implication that nothing happened by chance, Bernoulli, along with others at the time, believed that apparently random events, such as the fall of dice, were not contingent at all and their unpredictability was simply a consequence of ignorance (Hald, 1990, p. 248). Using this insight, he discussed the concept of 'moral certainty', which that had originated in Scholastic theology, and argued that a chance smaller than one in a thousand could be regarded as a 'moral impossibility' (Jorland, 1987). The next question is how one can achieve 'moral certitude'. The answer was provided by the Law of Large Numbers: repeat an experiment repeatedly until you have enough information to make a sound judgement. For Bernoulli, 'the Art of Conjecturing' came down to measuring, as accurately as possible, the level of imperfect knowledge – probability. The implication of this was that probability was fundamentally subjective since it relied on what personal knowledge (Hald, 1990, p. 250). Moreover, Bernoulli had taken a revolutionary step in applying mathematical probability to propositions, not just to events (Hald, 1990, p. 225). The final section is the most significant but has proved to be problematic for modern authors because Bernoulli considered situations where the sum of probabilities could be greater than one. This approach reflected the fact that he was working at a time when what was important was just treatment in financial contracts (Sylla, 2006, p. 28). It was not necessary that a probability summed to one, only unjust if it did not.

The explanation for the disappearance of the moral approach to probability disappeared is in the emergence in the early nineteenth century of what is known in Britain as 'Romantic Science' UK and '*Naturphilosophie'* in German. *Naturphilosophie* was personified by the naturalist, Alexander von Humboldt. Humboldt who sought an all-encompassing perspective that transformed a capricious nature into a cohesive whole (Daston, 2010). However, this attitude, with the scientist being part of, not an objective observer of, nature, implied that science was fundamentally subjective (Fara, 2009, pp. 215-218). To ensure that the ideas coming out of the mind of a scientist were true representations of the world, their observations had to be precise and accurate. In this context, Gauss developed the Normal distribution and the Central Limit Theorem to deliver a framework that addressed the fidelity of scientific observations.

A foundation of science today is to make repeated observations that are analysed using the tools of statistics resulting in an objective representation of the world. This scientific paradigm, synthesising Cartesian rationalism, German idealism and statistical empiricism came to dominate science from the second half of the nineteenth century. This reliance on observable facts has its limitations, since observable facts might not be available. This problem can be resolved through mathematics, which does not deal with facts but relations between objects (which might be facts), and aids natural science by enabling the interpolation of a theoretical fact out of empirical facts through a rigorous process of deduction and generalisation (Poincaré, 2001, pp. 110-111).

Truth in Financial Mathematics

The belief that science can establish an all-encompassing perspective that reveals calculable consequences out of random phenomena is the basis of most contemporary economic theory and reflects the Platonic idea that good government should be based on reason and foresight. In assuming the future is amenable to rational calculation, these theories fail to accommodate the 'inconceivable event'[1]. For example, many of the societal changes brought about by the introduction of smart-phone technology were inconceivable before 2003; as a result, financial predictions about the relative performance of Microsoft and Apple were, necessarily, wrong in 2000. Hence, outside the physical sciences, where stable relationships between objects exist, it becomes difficult to identify meaningful natural laws. This results in what Keynes described as 'radical uncertainty' in his *Treatise on Probability* (1921), or what has become known as 'Knightian uncertainty', a concept that emerged out of Knight's *Risk, Uncertainty and Profit* (1921).

In a financial market, a price is a statement about an uncertain future. As such, it is meaningless to assert that the price is a 'true' representation of an asset's value. Therefore the role of mathematics in finance is different to its role in the natural sciences. Mathematical theory in finance does not aim to guarantee that prices (statements) correspond to true values (facts). Rather, the Fundamental Theory of Asset Pricing requires a price has two important properties. Firstly, it ensures a price does not admit an arbitrage, the possibility (chance) of making a riskless profit. This echoes the view that profit can only arise out of uncertainty, fundamental in Aquinas' treatment of the Rhodean merchant as well as Knight's account of finance. Secondly, if all prices in a market preclude arbitrage opportunities, then all prices are consistent or coherent.

Ensuring the coherence of prices, but not their correspondence to asset values, means that prices might be internally consistent but never the less wrong. This is an example of Bertrand Russell's remark that "Bishop Stubbs was hanged for murder", a statement that, when Russell

[1] This is more serious than a 'Black Swan event'. Taleb's definition of an event is (i) it is outside normal expectations; (ii) it has significant impact; (iii) rationalised after the event. Chernobyl and the '9/11' attacks are 'Black Swans', but were also conceivable.

made it, was known to be false. Russell used the phrase to highlight that one could create a set of statements that were internally consistent but, none the less, wrong. This is the fundamental problem faced by a financial market, it is making statements that can bear no relation to facts yet are 'believed' in some sense. Russell argued (Zalabardo, 2015, pp. 7-9) that that beliefs must be based in 'something'. In this respect, The Fundamental Theorem of Asset Pricing, like all good mathematical theories, reveals the essential structure of markets. It identifies the 'something' that market participants believe in the concept of 'Justice' as delivered through the idea of 'reciprocity' − equality in exchange. This is the fundamental invariant in commerce manifested in the principle of 'no arbitrage' and ensures the objective validity of a price through the Fundamental Theorem of Asset Pricing (Johnson, 2015).

These observations on the role of reciprocity in contemporary mathematical theory do not directly address the problem of radical uncertainty. In *A Treatise on Probability*, Keynes had observed that in some cases cardinal probabilities of events could be deduced, in others, ordinal probabilities – one event was more likely than another – could be inferred, but there were a large class of problems that were not reducible to the concept of probability. This related to Aristotle's understanding that there were three classes of phenomena: events that were determined (the development of a bird embryo); those that were predictable (the weather) and those not amenable to science (the discovery of buried treasure). Keynes' argument was challenged by Frank Ramsey who, in *Truth and Probability* (1926), argued that probability relations between a premise and a conclusion could always exist (Ramsey, 1931), (Ramsey & Mellor, 1980), (Davis, 2004), (Edgington, 2012). He defined 'probability' as 'a degree of belief' and noted that a standard way of measuring these was through betting odds (Ramsey, 1931, p. 171). On this basis, Ramsey formulated the laws of probability (Ramsey, 1931, p. 181), finishing with the observation that

> These are the laws of probability, … If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have a book made against him by a cunning better and would then stand to lose in any event.(Ramsey, 1931, p. 182)

This is the 'Dutch Book' argument. In justifying his approach by saying if someone violated the rules then they would suffer is a version of the 'Golden Rule' (Wattles, 1996), (Hàjek, 2008) − "Do to others as you would have them do to you" (Leviticus 19:18, Confucius *Analects* XV.23-24, Matthew 7:12, Luke 6:31, Luke10:27, *Qur'an* 83.1-2) and is founded on the concepts of fairness and reciprocity, which are implicit in the mathematical theory.

Rather than insisting that a true statement must correspond to facts, Ramsey was employing a 'pragmatic' theory of truth where in "The opinion which is fated to be ultimately agreed to by all who investigate is what we mean by the truth" (Peirce, 1934, p. 407). In a radically uncertain environment, the truth of a statement − a price – cannot correspond to a verifiable fact − an asset's value.

Ramsey focussed on how a betting market can deliver the probability of a radically uncertain event. In doing this he did not describe what he meant by a market and, for clarity, a distinction has to be made between economic and financial markets. In economic markets 'brokers' act on behalf of property owners who have a clear appreciation of the value of the goods they own. Financial markets appear when the value of an asset is uncertain making it difficult for a broker to find property-owners who could agree a price. In these situations, 'jobbers'[2] or 'market-makers' trade amongst themselves and provide prices to agents outside the market, such as brokers. Jobbers are typically people with limited resources (Attard, 2000, pp. 13-14) (Mackenzie & Millo, 2001, pp. 19-22) who do not own assets long term.

Jobbers ensure that the market is in a position to offer an opinion/probability/price to the public through a discursive process, whereby one jobber makes a claim as to what the true price is and then the claim is challenged, a market seeks the pragmatic truth of a price. The recorded price is unlikely to be a mutually agreed price because a trade will only occur when someone believes the quoted price is wrong, and they can profit can by buying/selling at the price: silence is consent. A jobber might not quote the price they believed represented the value of the asset, since they might have a position that they want to close and so offer an attractive price. This implies that market prices will often reflect subjective, rather than objective, views and a quoted price does not have to correspond to a deductively, or inductively, proved judgement. Moreover, like a judgement delivered by a jury, it is not apparent how the jobber's opinion of the price is actually arrived at and so can never be clearly justified.

This process is a communal act and to be successful it must allow all statements to be challenged and defended without resorting to authority (Misak, 2002). What is important when a market-maker quotes a price is not that the price corresponds to the value of the asset being traded but that the market-maker is sincere in quoting the price. In recognition of this fact, the practice emerged in financial markets of jobbers being required to quote simultaneously 'bid' prices, at which they would buy an asset, and 'offer' or 'ask' prices, at which they would sell, without knowing if the counter-party is seeking to buy or sell the asset. This 'dual-quoting' requirement forces jobbers to be sincere in their pricing and emasculates any power they might have through accumulated wealth, since they would have to buy or sell at the prices quoted.

Jobbers avoid terms like 'buy' or 'sell' since they imply a commitment to an asset that would cloud judgement (Beunza & Stark, 2012, p. 394). While this disinterest in the asset is often

[2] The role of jobbers was an established part of the British financial system from the late eighteenth century until 1986 when they lost their distinctive status in the 'Big Bang' reforms.

perceived as cynical, it is none the less sincere since the jobber's "manifest intention is meant as it is expressed" (Habermas, 1985, p. 99) because they will be required to act on the prices they quote.

Jobbers generate financial markets by making assertions as to the price of an asset in a radically uncertain environment. Traders challenge these statements by buying or selling at the price offered. An individual's beliefs can only be confirmed, or refuted, through discussion with others. This highlights that markets are primarily concerned with a community converging on agreement. Realising that reciprocity is embedded in the mathematics of markets while sincerity is intrinsic in the rules governing market-making implies that financial markets are not competitive arenas, where those blessed with a peculiar capacity for rational foresight thrive. Rather, it suggests that markets work well as places where opinions − as to the value of assets − are discussed; that they are 'centres of communicative action'.

Pragmatism argues that whether in ethics, mathematics or language, norms – rules that guide behaviour – emerge out of practice and become formulated as explicit rules or principles because they work (Brandom, 1994, p. 21). On this basis, the norms necessary for a community to converge effectively on understanding (Habermas, 1984, p. 99) have been explored. Statements need to be comprehensible and, where appropriate, conform to matters of fact. They must be objectively true. Statements must also be the honest intention of the speaker; they must be truthful. Finally, they must conform to what the community believe is right and be ethically or morally acceptable. These conditions need not be simultaneously satisfied. A statement should not be accepted because it conforms to social convention of what is right if a matter of fact is clear in denying its objective truth. Alternatively, a matter of fact cannot pertain to uncertain domains, where the rightness of a statement might dominate.

Since market pricing relates to a radically uncertain future or, equivalently, situations that are so complex that not all the interconnecting facts can be gathered in the time available, it is a problem of *practica*. In these circumstances, the objective validity of a price – its factual truth – only exists in so far as the price ensures reciprocity; *eadl* (عدل); *gong zheng* (公正). Reciprocity is essential because it delivers justice in exchange that supports social cohesion. When faced with radical uncertainty, the subjective validity of a price in a market is addressed by the norm sincerity, *ikhlas* (إخلاص); *xin* (信). It does not ensure that a subjective opinion is objectively true but ensures that people mean what they say and are honest in their intentions. In markets, they must 'put their money where their mouth is'.

While market sincerity is manifested in the statutory status of market-makers and reciprocity is embedded in mathematical theory, it is harder to identify clear evidence for social 'rightness' being explicit in market discourse. However, since financial markets are concerned with discovering prices in a complex and uncertain environment, when it is impossible for a price to correspond to a matter of fact the 'rightness' of prices needs to be addressed. In the context of finance, charity, or benevolence, comes closest to capturing the idea. This point is demonstrated in Shakespeare's *The Merchant of Venice*. While a popular play, *The Merchant* is regarded as problematic (Midgley, 1960, p. 119). Many contemporary audiences find the play incoherent and the last act of the play, coming after the drama of the trial scene, is thought redundant. However, if the play is interpreted in the context of Renaissance humanism synthesising classical philosophy and Biblical allusion, the play becomes coherent and highlights the necessity of charity in (Christian) finance (Gollancz, 1931), (Coghill, 1950), (Lewalski, 1962). The basis of this interpretation is that the play is a study of the four types of classical love: *storge* - familial love; *philia* - friendship; *eros* - physical love; and *agape* - spiritual love. In this context, Antonio, the 'merchant of Venice' personifies *agape*/charity/*caritas*, the principle characteristic of Christ and is central to the Augustinian doctrine that directs western Christianity.

The Scholastics argued that usury was prohibited because it involved taking more than what had been given and so the equality necessary for economic exchange to promote social cohesion was lost. Shakespeare takes this line and argues that usury alienates members of a community from each other, while mercy brings them together; he makes a similar point in *Measure for Measure* (Dickinson, 1962), (Wilson, 1994). In *The Merchant*, Shakespeare highlights the essential importance of *caritas*/*agape*, divine love, in Christianity. In contrast to Judaism, and Calvinism, the play is advocating that judgements should not be based solely on 'the Law', which as Calvin pointed out would permit usury (Wykes, 2003, pp. 41-44), but also on mercy (Coolidge, 1976, p. 256).

This point had been made by William Tyndale who wrote in the introduction to his translation of the New Testament that the law, "through teaching every man his duty, doth utter our corrupt nature" and "only love and mercifulness understandeth the Law, and else nothing" (Coolidge, 1976, p. 246). In the context of pricing, reciprocity is embedded in the mathematical 'laws' that provide the only objective measure in an uncertain world. This objective measure needs to be supplemented by subjective sincerity and social charity for markets to work well in supporting social cohesion.

The norms reciprocity, sincerity and charity, work together to inhibit the emergence of inequality. This is important as equality within a community ensures that a plurality of views can be expressed on a topic. A plurality of sincere views is required so that the best opinion can emerge (Misak, 2002, pp. 127-147). In financial markets, as well as through jobbers, the plurality of opinions has been has been delivered through phenomena such as *ducaton* shares (Poitras, 2000, pp. 276-277) and bucketshops (de Goede, 2005, pp. 68-71), which enable people without much property to challenge the opinions of property owners but needs them to have 'skin in the game' (Taleb & Sandis, 2014).

Evidence for the relevance of norms to finance is given by the experience of the Quakers. The 'Quaker success story' in finance was built on the fact that they were trustworthy when all around them there were "usurious contracts, false chevisance and other crafty deceits" (Murphy, 2009, p. 83). Trust is defined as "a firm belief in the reliability, truth or ability of someone" (Oxford English dictionary). Accounts of how trust is developed vary, however they involve terms connected to sincerity, such as: honesty, integrity, credibility, predictability, dependability and reliability; terms connected to reciprocity, such as: judgement and fairness; and terms related to charity, such as: benevolence, goodwill and responsibility (Seppänen, Blomqvist, & Sundqvist, 2007).

Financial Crises

One feature of the Credit Crisis of 2006–2009 was that the instruments at the heart of the Crisis, Collateralised Debt Obligations (CDOs), were not priced using the standard market mechanism based on the dual-quoting of jobbers (MacKenzie, 2011, p. 1780). Prices were based on ratings given by agencies paid by the producer of the financial instruments (a 'broker-mediated model') and mathematical models using parameters based on unrelated markets. This illuminates that there are two types of financialisation, the phenomenon where "profits accrue primarily through financial channels rather than through trade and commodity production" (Sahlins, 2003, p. 282), (Krippner, 2005, p. 174) which is made possible by financial contracts. The first type of financialisation involves market-makers and speculators pricing in a discursive manner through the application of sincere, subjective judgement. The second involves brokers supported by 'quants' employing algorithms and data – and claims to be objective judgement – in a strategic manner. The second form of financialisation – as most experienced traders believed it would (Tett, 2009), (Haug & Taleb, 2011), (Duhon, 2012) – failed. From a mathematical point of view, models failed because they were being regarded as representations of the future, rather than signifiers. The legitimate use of mathematical models is to develop a clearer understanding of what can be inferred about market sentiment from jobber-mediated market prices so that investment decisions can be taken (Johnson, 2011), (Beunza & Stark, 2012, pp. 384-385), (Duhon, 2012, pp. 265-277).

These observations reveal the general principle that when a mathematical model is applied to radically uncertain situations, it cannot be used to deliver representations that correspond to the future: to predict. In such situations, mathematical models are best employed in a similar manner to the way architects and engineers use models, to simplify complex systems and enable people to come together, discuss the system, tease out the significant issues and form a consensus (MacKenzie, Muniesa, & Siu, 2007).

In uncertain finance, prices signify value (Muniesa, 2007) (Muniesa, 2011) they cannot correspond to value. Financial engineers realised this in the aftermath of the 'Black Monday' crash of October1987. Earlier in the year, the Black-Scholes-Merton (BSM) options pricing formula had been described as the most successful model in economics (MacKenzie D. , 2008, p. 177). This claim had had been based on detailed analysis of data on option prices in the market compared to the predictions of the model. Before around 1976, traders priced options using intuition and judgement. After 1976, prices began to converge onto the theoretical values predicted by the BSM model. One argument is that the traders began to adopt the hedging-strategies and, in doing so, the options became 'correctly' priced. An alternative explanation is that a subtle feedback loop was taking place: the traders believed that the model prices were correct and so they moved towards the model prices. This, in turn, reinforced traders' faith in the models. Markets began to 'perform' the financial theory (MacKenzie D. , 2008, pp. 243-275) and as they did, they became disconnected from the reality of the economy.

The market crash of Black Monday shattered the faith of traders in the option pricing models (Miyazaki, 2007, pp. 409-410), (MacKenzie D. , 2008, p. 248), (Haug & Taleb, 2011). Academics had already realised that a weakness of the BSM model was that it required an estimate of the volatility, the parameter that determined the variance of future asset prices. This assumed that the economy is ergodic, and volatility could be estimated from the history of asset prices. This is difficult to justify and modellers responded by introducing random volatility into their models (Hull & White, 1987). However, the random volatility was itself assumed to be ergodic. Models were becoming more complex rather than tackling the essential issue of radical uncertainty.

After October 1987, it was observed that the market prices of options did not reflect the model prices in a consistent way, manifested in the 'volatility smile' or the 'volatility skew'. These phenomena were caused by traders anticipating greater price moves than those predicted by the BSM model. As a result, the BSM model was used not to price options, but to extract information on the market's views on volatility from the options' prices. In 2004, this was captured in a description of trading

> A few desks down [a trading floor] is a math Ph.D. from Cambridge. He spends much of each day studying the fast-changing "volatility surface" of the options market – an imaginary 3-D graph of how price fluctuations widen and narrow as the terms of each option contract vary. … it is a wild, complex shape. (Mandelbrot & Hudson, 2005, p. 81).

BSM had ceased to calculate prices from probabilities, as post-Laplacian classical probability theory would suggest. Rather it determined volatility – essentially the market's view of the probability distribution of asset prices in the future – from prices. This mimicked the approach taken by the early pioneers of probability in the seventeenth century.

Further evidence of using financial models as signifiers to highlight where there might be issues that need exploring, rather than delivering a true representation, is given by the renaissance of the bank J.P. Morgan's immediately following the Credit Crisis (Tett, 2009, pp. xiii-xiv,289-291).

In the period 2005–2006, J.P. Morgan's shareholders were putting the bank's managers under intense pressure to mimic the growth in revenues being reported by other investment banks who were actively investing in CDOs of MBS (Tett, 2009, pp. 143-148). J.P. Morgan's competitors were reporting the large revenues based on the 'objective truth' represented by the Gaussian copula model. Rather than place all their trust in the model, J.P. Morgan's mathematicians reverse-engineered the model to see what assumptions were being used to generate the attractive prices of MBS CDOs. They realised that the profits relied on the assumption that the correlation parameter, rho, was 0.3, based on the correlation between corporate bond defaults. The bankers – working closely with the mathematicians – 'sense checked' this and could not see how this choice of rho could be justified in the case of sub-prime mortgages; if someone in a street was unable to make their payments there was a high probability that their neighbour was struggling as well (Tett, 2009, pp. 148-151). By interrogating the model, they deduced that the attractiveness of CDO of MBS was based on an unfeasibly optimistic choice of rho. J.P. Morgan had used the Gaussian copula as a signifying model that helped clarify what was important in the market, not as an oracle that magically revealed a hidden truth of the market.

This 'reverse engineering' of both BSM and the Gaussian Copula reflect an understanding that there are no concrete facts against which pricing models can be tested. The differences between the pre-1987 and post-1987 approaches to BSM and the approach J. P. Morgan and that taken by most investment banks to the Gaussian before 2006 can be understood in terms of the distinction between rationalism and empiricism in science.

Descartes developed his rationalism in an attempt to establish 'certain knowledge' by developing a way of ensuring that the mind held true representations of the world. For Descartes, the mind interacted with material bodies through 'ideas' (Nadler, 2006) and the problem of certainty came down to ensuring that the mental representations, perceived through the unreliable senses, accurately corresponded to the physical world. Thinking is a personal process, which implies it is subjective and so to ensure the objective truth of ideas, Descartes resorted to invoking a benevolent God, who does not deceive, to guarantee the reliability of his thinking mind.

Spinoza developed Descartes' work by collapsing the three types of substance Descartes' employed: matter, mind and God, into one. This was captured in his phrase *Deus sive natura*, 'God or nature', meaning that there is only a single substance (Spinoza, 2002, p. 224) that, when viewed from one perspective is nature but from another is God. This solved the problem of how Descartes' mind interacted with matter at the cost of prohibiting contingency (Spinoza, 2002, p. 232) because if everything was connected to God, it could not happen by chance. This also meant that emotions were not part of the mind, and so could not be rationalised, but were governed by the laws of nature (Spinoza, 2002, pp. 277-278). These ideas became an important influence on German idealism and hence *Naturphilosophie* / 'Romantic science' (Frank, 2003), (Förster & Melamed, 2012).

In contrast to Cartesian rationality there is the empirical approach introduced by Locke. Unlike Descartes and Spinoza, Locke argued that human knowledge could never be certain. Knowledge came from experience; sense organs first perceived events in the real world and then the mind interpreted them to form an idea. The validity of an idea depended on its origins and how the idea had evolved. This entailed that people investigate how their beliefs came about, their genealogy. Descartes described knowledge as being like a tree, its roots were in metaphysics while its trunk was made up of mathematics – "on account of the certitude and evidence of [its] reasoning" (Descartes, 2008, p. Part I)– and physics, with the branches of the tree being the practical sciences, both natural and moral. Locke took a different position and divided knowledge into three types (Locke, 1690, p. Chapter 21). *Physica* was concerned with the nature of things, what is in the world; *practica* related to what people should do as rational and wilful agents; while *semeiotika* referred to how *physica* and *practica* are attained and communicated.

Descartes' philosophy can be caricatured as being about ensuring certainty when there was doubt about what was true in a particular situation. Spinoza developed this by seeking a viewpoint that ensured the truth could be discerned. These two approaches were fundamentally theoretical while Locke offered a philosophy grounded in what people did in practice. Locke implicitly accepted that people, having had different experiences, would have different viewpoints resulting in multiple interpretations of a single situation. What was essential was that a belief accurately represented the synthesis of experiences and the matter at hand – it was sincere – and Locke can be characterised as focusing on how a belief could be trusted. Locke claimed that trust was the glue that bound a society together (Locke, 1954, pp. 213-214) and that language was important because it enabled promises to be made.

These observations reveal that financial mathematics is a manifestation of Locke's empiricism rather than Cartesian rationalism. The explanation is that as a radically uncertain environment, the pursuit of certainty, central to rationalism, is futile.

Insights

Aristotle distinguished *episteme* − passive knowledge − from *phronesis* − active thinking and the antonym of "false belief" is not "true belief" but "justified belief" and science should focus on the manner of its justifications rather than its results. Re-emphasising *phronesis*, with the aim of good, virtuous, living, is an insight that has emerged out of our experience of the Financial Crisis, which highlighted that scientific knowledge is not as robust as many of us would like to believe. For example, most of the research in Financial Mathematics has been focused on establishing the "true" price of a contract, rather than exploring the principles that make thinking robust, given that we cannot rely on certain knowledge in an uncertain world. It is clear that mathematics and statistics are crucial to our understanding of the world. But all too often students learn to master quantitative methods without ever discussing if and why they should be used, the choice of assumptions and the applicability of results. Mathematical models are too often employed to obfuscate rather than inform, as described the Bank of England's testimony to the Parliamentary Commission for Banking Standards

> unnecessary complexity [of mathematical models] is a recipe for […] ripping off […], in the pulling of the wool over the eyes of the regulators about how much risk is actually on the balance sheet, through complex models.

The existence of a model was enough to close down debate and discussion, there was little investigation of how it worked; how it justified. This analysis is barren unless tangible insights can be drawn from it.

Credit scoring, the basis of most commercial lending decisions, is an established topic of mathematics. From a practical perspective, there is significant uncertainty as how to handle the results of credit scoring models. They typically seek to deliver guidance on a loan decision based on a variety of data sources. However, it is observed that the decisions from different models used by a single institution are frequently contradictory. These differences are often understood by expert modellers but can lead to doubt in the minds of decision-makers, who believe a model should present an objective representation of the credit worthiness of the borrower. A response to this uncertainty is to build more complex models that integrate more statistical data, rather than accept the differences and make a judgement.

An alternative to building ever more accurate representational models is to view the models as signifiers. Each model presents a slightly different perspective on an unknowable future. In this conception a collection of models represents a ‘college’ rather than a ‘toolbox’ and the decision-maker must integrate the different perspectives to make a judgement. This is possible through deliberation, a topic of philosophical rhetoric, even when views are based on implicit intuition and not explicit deduction. This is important outside finance where results of machine learning and agent based modelling are often not amenable to audit. Establishing the rhetorical conventions that can accommodate the intuition of machines, just as we can accept the intuition of humans in deliberation, would be revolutionary. This relates to questions about the deployment of AI within an organisation (Tarafdar, Beath, & Ross, 2017) and how to make AI more trustworthy (Lei, Barzilay, & Jaakkola, 2016).

Algorithmic pricing is becoming ubiquitous. Not only is it a significant feature of brokering on financial exchanges but it is becoming a feature of retail markets (Chen, Mislove, & Wilson, 2016). Algorithmic trading can improve competitiveness and efficiency in a market, but it can also have unintended consequences ('flash crashes', products being offered at extraordinary prices) or might even be designed to manipulate markets to the benefit of the algorithm user. The crude approach that all market activity is beneficial (Foresight, 2012, p. Section 8.2) pre-supposes that liquidity is a utility service and not a more complex consequence of market trust. Historically, market liquidity was provided by market-makers or jobbers who were obliged to simultaneously offer prices at which they would both buy and sell. This 'dual quoting' was designed to ensure the market-maker was sincere in their pricing. On this basis a counterparty could trust the price they were being offered and highlights the significance of the subjective dimension in establishing trust. The project could investigate in what sense can an algorithm, or any automated system, can be considered to exhibit subjective and socially acceptable judgements and how this could be assessed by regulators.

Trust in Bitcoin, and other crypto-currencies, is founded on the block-chain and the way Bitcoins are minted. These algorithms are supposed to offer an objective basis for trust in the currency that circumvent the need for subjective and social foundations (Christopher, 2016). However, one view of Bitcoin is that it is a confidence trick designed to make its inventors incredibly rich off the huge reservoir of criminal greed and the paranoia of the alienated[3]. The success of Bitcoin is built on 'objective' criteria that give a misplaced perception of privacy (they leave a digital trail that can only be disrupted through the use of 'tumblers'), reliability (transactions are dropped when traffic is high) and security (an agent who controls 50%+1 of the nodes, controls the ledger). Money traditionally had to be universal – it is used by both princes and paupers – and fungible. With the advent of crypto-currencies this is changing and presents a risk that inequalities within society are reinforced and the communality of money is lost.

Trust is also an essential component of crowdfunding facilitated by digital platforms. The platform acts as a broker and, for the sake of efficiency, the key credit rating and pricing processes are automated. There is a risk, identified in finance, that algorithmic decision making can result in 'super-portfolios' that give the illusion of diversification that results in crashes (MacKenzie, 2003). With the advent of algorithmic trading, the issue of 'unthinking' algorithms interacting has emerged. This echoes the phenomenon outside finance of polarisation on social platforms. The issue these phenomena raise is related to justification and legitimisation underpinning trust.

In the financial markets, it is important to provide a post-hoc explanation for a decision. A significant limitation of many algorithmic systems (for instance, those which employ neural networks) is the difficulty in delivering an auditable, deductive chain of reasoning that resulted in a decision. However, the reality of human judgement, is that decisions are implicit and are

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[3] Kaminska, *What is crypto's agenda really*, https://ftalphaville.ft.com/2017/08/23/2192693/what-is-cryptos-agenda-really/, 23/08/2017

justified *post-hoc* through deliberation; pragmatism that argues knowledge resembles a cable of thin interweaving strands rather than a chain of strong links that is vulnerable to a single link failing.

Society, in general, is struggling to reach a consensus on a range of topics. The failure of rationally based climate science to deliver radical policy actions can be explained in terms of two polarised groups failing to explore the genealogy of their disagreements. The UK's EU referendum focussed on disagreements about financial claims of the future rather than the significance of the substantive choices being made.

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